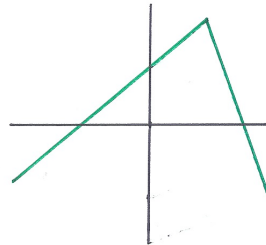
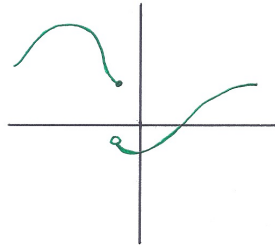


## Graphs of Polynomial Functions

1. The graph of a polynomial function is continuous ; the function has no breaks, holes or gaps.
  - a. has only smooth, rounded turns
  - b. examples that are not polynomial functions

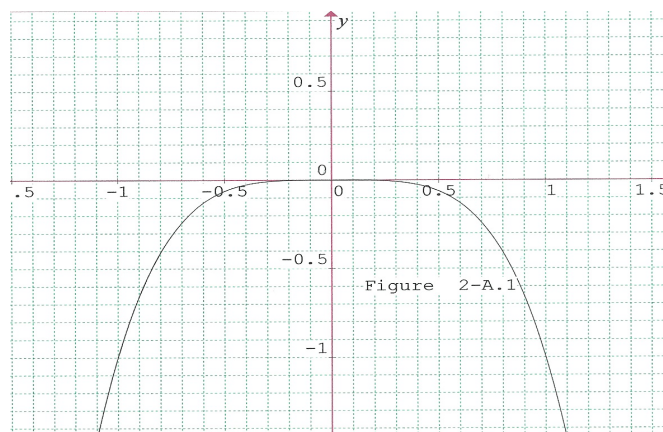


2. simplest polynomials are monomials of the form  $y = f(x) = x^n$

## 3. examples : transformations of polynomial functions

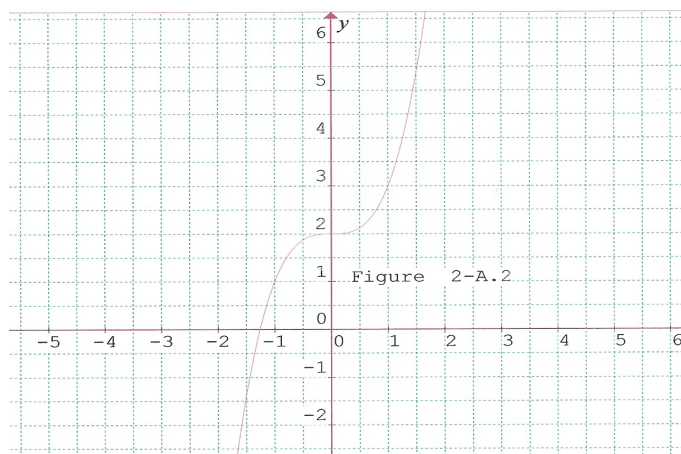
(a)  $f(x) = -x^4$

⇒ the degree is even, so it is similar to the parabola  $y = x^2$  ; the negative coefficient reflects the graph in the x-axis



(b)  $y = x^3 + 2$

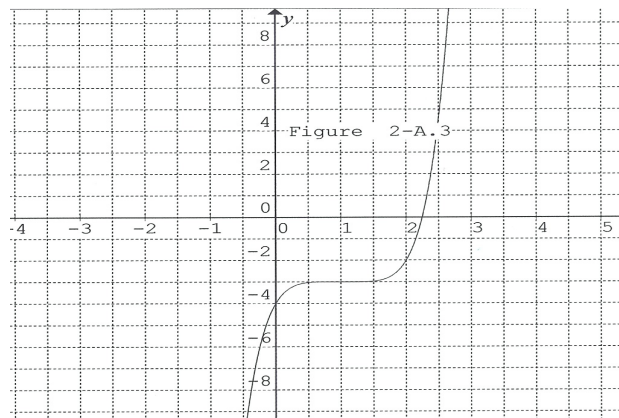
⇒ this is an upward shift of 2 units to the graph of  $y = x^3$



(c)  $g(x) = (x - 1)^5 - 3$

→ this represents a shift left of 1 unit and down 3 units of the graph

$$g(x) = x^5$$



4. Leading Coefficient Test - whether the graph rises or falls can be determined by the degree of the highest term (even or odd) and by its leading coefficient (number with the variable of the largest degree). The rules are as follows, and together, they make up what's called the Leading Coefficient Test (LCT)

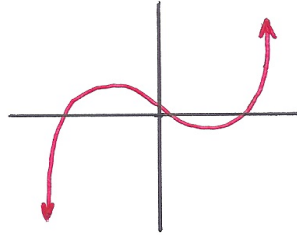
Given  $f(x) = ax^n$

$n$  → exponent

$a$  → leading coefficient

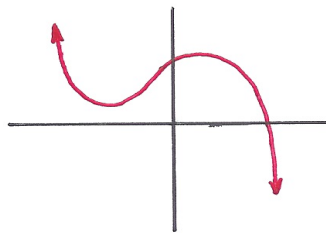
- (a) If  $\underline{n}$  is odd and  $\underline{a}$  is positive .... graph is down Left and up Right

Figure 2-A.4



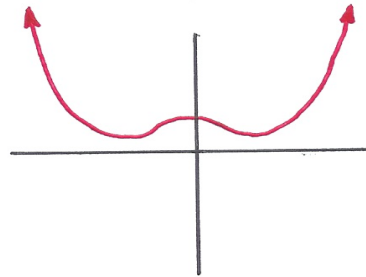
- (b) If  $\underline{n}$  is odd and  $\underline{a}$  is negative ..... graph is up Left and down Right

Figure 2-A.5



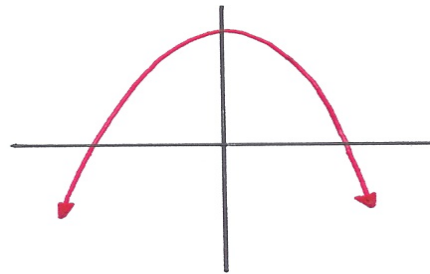
- (c) If  $n$  is even and  $a$  is positive ..... graph is up Left and up Right

Figure 2-A.6



- (d) If  $n$  is even and  $a$  is negative ..... graph is down Left and down Right

Figure 2-A.7



## 5. Examples using the Leading Coefficient Test

(a)  $f(x) = -8x^5 - 7x^3 + 11x$

(b)  $g(x) = 3x^2 + 6x - 9$

(c)  $h(x) = -4x^4 + 10x^7$

Answers: (a) up left, down right  
(b) up left, up right  
(c) down left, down right

6. Real zeros of a polynomial - all of the following statements are true  
[If  $f$  is a polynomial function and  $b$  is a real number]

- (a)  $x = b$  is a zero of the function  $f$   
(b)  $x = b$  is a solution of the polynomial equation  $f(x) = 0$   
(c)  $(x - b)$  is a factor of the polynomial  $f(x)$   
(d)  $(b, 0)$  is an x-intercept of the graph of  $f$

## 7. Finding the zeros is one of the steps that should be taken when graphing a polynomial function ; however, you may not always be able to find the zeros of a function.

8. When a graph changes direction (from decreasing to increasing or visa versa) it is said to have a relative minimum or relative maximum at that point . A polynomial function will change directions one less time than its degree.

ex)  $f(x) = 5x^6 - 3x^4 + 2x^2 - 5$

degree = 6 , therefore, changes direction  
5 times

9. Finding the value of k

- (a) If k is odd, the graph crosses the x-axis at  $x = a$   
 (b) If k is even , the graph touches the x-axis (but does not cross) at the point  $x = a$   
 (c) this situation (from part b) occurs when there are repeating zeros. For example, in the function

$$f(x) = -3x^4 + 3x^2$$

$$-3x^4 + 3x^2 = 0$$

$$-3x^2(x^2 - 1) = 0$$

$$-3x^2(x - 1)(x + 1)$$

\* the real zeros are  $x = 0$ ,  $x = 1$  and  $x = -1$  ; however, at  $-3x^2 = 0$  , there is a repeated zero (because of the second power) ;

The value of  $k$  is the exponent when the polynomial is in complete factored form. So, for  $-3x^2$ ,  $k = 2$ , for  $(x - 1)$ ,  $k = 1$  because the number outside the parentheses is understood to be one; same with  $(x + 1)$ . Therefore, the graph would cross the  $x$ -axis at  $x = -1$  and  $x = 1$ , but it would only touch at  $x = 0$ , because the value of  $k$  is even.

**Homework Problems** ..... *check your answers on graphing calculator*

Graph each polynomial function without using your graphing calculator. Use the LCT, find the  $x$ -intercepts, state the number of times the curve changes directions, and find the value of  $k$  for each solution. Make a sketch and then check it. Here is a good online graphing calculator (<http://webgraphing.com/index.jsp>). Choose graphing from the horizontal menu, and then 'functions', and then type in your function.

1.  $f(x) = x^4 + 2x^3$

2.  $g(x) = x^3 - 9x$

3.  $g(x) = x^4 - 4x^2$

4.  $f(w) = w^5 - 6w^3 + 9w$

5.  $f(x) = x^2(x + 3)$

6.  $y = -2x^2 - 5x$

7.  $g(x) = -\frac{1}{4}x^4 + 2x^2$

8.  $f(x) = x^4 - x^3 - 20x^2$

